Structured credit risk

Allan M. Malz

Columbia University

Structured credit risk analysis

Structured credit risk measurement

Structured credit risk analysis
   Securitization loss scenarios
   Securitization and leverage

Structured credit risk measurement

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Waterfall and tranche cash flows under loss scenarios

- Waterfall begins with *underlying collateral*
  - Loan proceeds (as multiple of notional) depend on default rate x:
    $$\text{loan proceeds}(x) = (1 - x)(1 + r_l), \qquad 0 \leq x \leq 1$$
- *Senior bond* has priority claim over mezzanine and equity
  - Receives all loan proceeds up to its own par value and coupon $(1 - a_s)(1 + c_s)$:
    $$\text{senior cash flow}(x) = \min[(1 - x)(1 + r_l), (1 - a_s)(1 + c_s)]$$
- *Mezzanine bond* paid only if senior bond paid in full
  - Receives all post-senior loan proceeds up to its par value and coupon $(a_s - a_m)(1 + c_m)$:

  mezzanine cash flow(x)
  $$= \max[\min[(1 - x)(1 + r_l) - (1 - a_s)(1 + c_s), (a_s - a_m)(1 + c_m)], 0]$$

- *Equity* receives remainder, if positive

  equity cash flow(x)
  $$= \max[(1 - x)(1 + r_l) - (1 - a_s)(1 + c_s) - (a_s - a_m)(1 + c_m), 0]$$

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Tranche returns under loss scenarios

- Compute tranche returns as a function of loan default rate from tranche cash flows and "thickness"

$$\text{senior return}(x) = \frac{\text{senior cash flow}(x)}{1 - a_s} - 1$$

$$\text{mezzanine return}(x) = \frac{\text{mezzanine cash flow}(x)}{a_s - a_m} - 1$$

$$\text{equity return}(x) = \frac{\text{equity cash flow}(x)}{a_m} - 1$$

- We can measure the return to each tranche in stress scenarios
  - E.g. x far in excess of π

	senior	mezzanine	equity
Baseline ($x = 0.05$) cash flow ($ mio.)	83.600	16.275	5.575
Baseline ($x = 0.05$) return (%)	4.50	8.50	11.50
Maximum ($x = 0$) return (%)	4.50	8.50	122.50
Stress case ($x = 0.125$) return (%)	4.50	−9.83	−100.00

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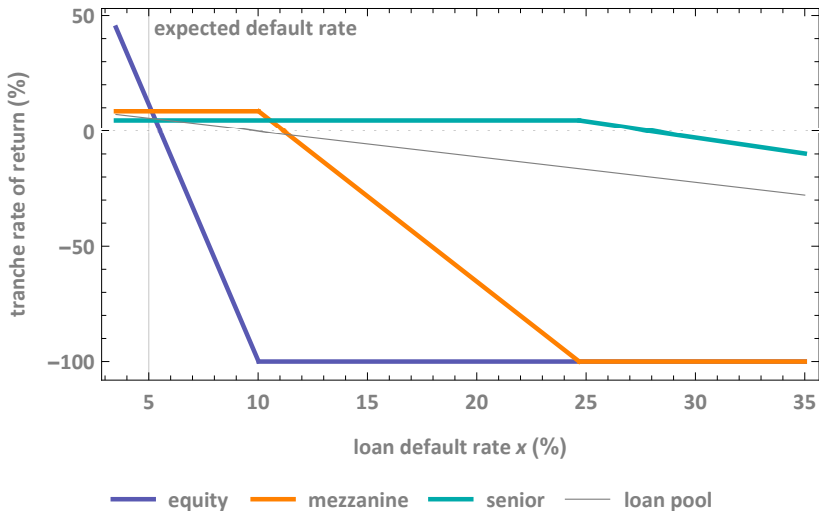 └─Securitization and leverage

Tranche returns are option-like

- Securitization tranches behave like options on underlying loan pool credit losses/proceeds
- Strike levels: attachment/detachment points
  Senior tranche behaves like a "short call" on loan pool proceeds
  Mezzanine tranche behaves like a "collar" on loan pool proceeds
  Equity tranche behaves like a "long put" on loan pool proceeds
- Payoff profiles and exercise prices defined in terms of loss levels at which the bond tranches default
- Leads to market risk behavior driven by changes in expected default rate and default correlation

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Tranche thickness and leverage

- Junior securitization tranches (mezzanine and equity) contain
  embedded leverage
- Thin tranches take proportionally greater losses for a given pool loss
  rate
- Tranche suffers losses only once its attachment point is breached
- Embedded leverage thus generated by two characteristics
  - Tranche thinness in conjunction with
  - Low position in waterfall
- For example, a 10 percent loan default rate
  - Barely brings pool rate of return to zero
  - But leads to total loss on equity tranche

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Pool and tranche returns in the example

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Default of a bond tranche

- Event of default of a bond tranche defined similarly to non-securitization bond: failure to pay principal or interest due
  - In our example, bond default occurs only at single one-year payment date for bond principal and interest
  - Insolvency may become evident well within one year, e.g. if realized loan defaults high
- Find default-triggering loss level x°:

$$\left.\begin{array}{r} \text{senior} \\ \text{mezzanine} \end{array}\right\} \text{defaults}$$

$$\Leftrightarrow (1-x)(1+r_l) < \left\{ \begin{array}{l} (1-a_s)(1+c_s) \\ (1-a_s)(1+c_s) + (a_s - a_m)(1+c_m) \end{array} \right.$$

  - Tranche with default-triggering loss level x° defaults if $\tilde{x} \geq x^\circ$
  - Probability of tranche default is $\mathbf{P}\left[\tilde{x} \geq x^\circ\right]$
- If senior bond defaults, mezzanine bond must also default
- Equity tranche cannot default
  - But can suffer lower-than-expected/negative returns

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Default and distressed returns in the example

- Default-triggering loss level x° for bond tranches:

$$x^\circ = \left\{ \begin{array}{l} 1 - \dfrac{(1-a_s)(1+c_s)}{1+r_l} \\[3mm] 1 - \dfrac{(1-a_s)(1+c_s) + (a_s - a_m)(1+c_m)}{1+r_l} \end{array} \right\}$$

for the $\left\{ \begin{array}{c} \text{senior} \\ \text{mezzanine} \end{array} \right\}$ tranche

	senior	mezzanine
x° (%)	24.685	10.023

- Loss level x at which equity tranche return is zero:

$$1 - \frac{(1-a_s)(1+c_s) + (a_s - a_m)(1+c_m) + a_m}{1+r_l}$$

or 5.518 percent

- Loss level at which equity tranche is wiped out (return $= -100$ percent) is identical to mezzanine default-triggering loss level

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Valuation and risk modeling approaches

- Risk analysis of securitization tranches based on risk analysis of underlying loan pool
- Typical rating agency approach: credit stress scenarios
  - "What-if" scenarios featuring much higher-than-expected default rates
  - Stipulate default and recovery behavior of the loan pool over time
  - Trace through cash flow results and effects on each tranche
  - Determine loss levels that "break" each tranche
- Formal credit portfolio modeling approaches
  - Simulation approaches, generally using ($\rightarrow$)copula models
  - Specific credit models, e.g. single-factor model

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Credit risk analysis of a securitization

- Assume collateral pool highly granular
- Combine risk analysis of loan pool with securitization waterfall to analyze credit risk of any securitization tranche
- Risk analysis based on credit loss distribution of tranches
- Explore impact of change in default probability and of high correlation

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Applying single-factor model to example

- Distributions of pool losses/tranche returns depend on market factor
  - Given assumptions on expected default rate (π) and correlation to market factor (β)
  - Default correlation among loans is β^2
- Cumulative distribution function of pool losses (a random variable $\tilde{x}$) in single-factor model:
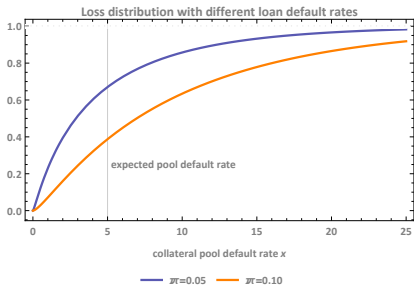
$$\mathbf{P}\left[\tilde{x} \leq x\right] = \Phi\left[\frac{\sqrt{1-\beta^2}\Phi^{-1}(x) - \Phi^{-1}(\pi)}{\beta}\right]$$

- Assume securitization constructed under baseline parameters
- Study effect on credit loss distributions of varying π and β
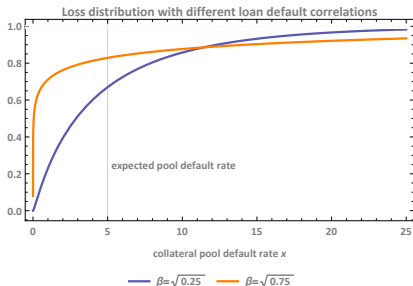- Baseline parameters:

$$\pi = 0.05$$
$$\beta = \sqrt{0.25}$$

Cumulative distribution function of pool losses

Loss distribution with different loan default rates

Loss distribution with different loan default correlations

Both plots use baseline parameter
$\beta = \sqrt{0.25}$.

Both plots use baseline parameter
$\pi = 0.05$.

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Correlation and collateral pool losses

- Default correlation has large impact on risk of equity and senior tranches
- Higher default correlation→higher likelihood of default clusters
- Loss distribution becomes skewed
- →Higher tail risk, i.e. likelihood of *both*
  - Very large losses
  - Very small losses

	$\beta = \sqrt{0.25}$	$\beta = \sqrt{0.75}$
$\mathbf{P}\left[\tilde{x} \leq 0.01\right]$	0.230	0.711
$\mathbf{P}\left[\tilde{x} \geq 0.25\right]$	0.017	0.066

Tranche risk analysis

- Each scenario/realization x of pool defaults has
  - Probability $\mathbf{P}[\tilde{x} \leq x]$
  - Waterfall→cash-flow consequences for each tranche
- →Cumulative distribution function of cash flows for each tranche
  - For example, cash flow CDF of the senior tranche is the set of pairs

    $$\{\text{senior cash flow}(x), \mathbf{P}[\tilde{x} \leq x]\}, \quad x \in [0, 1]$$

  - Can be computed for all tranches
- Can be mapped into CDF of returns as well as cash flows for each tranche

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Probability of default of a bond tranche

- Probability of default of a bond tranche can be computed via loss distribution function
- We have calculated default-triggering loss level for each bond tranche based on its coupon and the waterfall
- Tranche with default-triggering loss level x° defaults if pool losses reach or exceed that level: $\Leftrightarrow \tilde{x} \geq x^\circ$
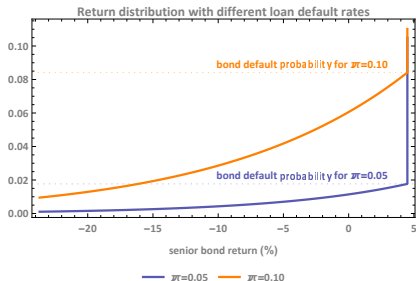- $\Rightarrow$ Probability of tranche default is

$$\mathbf{P}\left[\tilde{x} \geq x^\circ\right] = 1 - \mathbf{P}\left[\tilde{x} \leq x^\circ\right] = 1 - \Phi\left[\frac{\sqrt{1-\beta^2}\Phi^{-1}(x^\circ) - \Phi^{-1}(\pi)}{\beta}\right]$$

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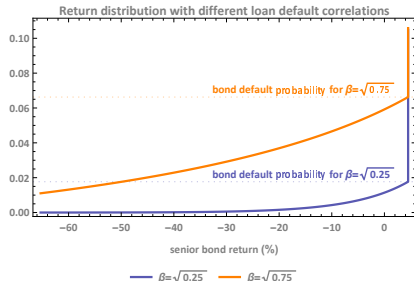
Risk analysis of senior bond

- Higher pool loss default rate and higher default correlation both bad for senior bond
  - I.e. shift return distribution function to left
- Risk of senior bond is very sensitive to default correlation
  - With correlation very low, senior bond default probability low even with high pool default rate
- High default correlation induces higher probability of default clusters that can reach into senior tranche
- Table displays default *probabilities* for different settings of pool loss distribution *parameters*

Senior bond default probability			
	$\beta = \sqrt{0.05}$	$\beta = \sqrt{0.25}$	$\beta = \sqrt{0.50}$	$\beta = \sqrt{0.75}$
$\pi = 0.025$	0.0000	0.0031	0.0184	0.0309
$\pi = 0.05$	0.0000	0.0177	0.0503	0.0663
$\pi = 0.10$	0.0030	0.0842	0.1297	0.1390

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Pool default behavior and senior bond returns

Return distribution with different loan default rates

bond default probability for π=0.10

bond default probability for π=0.05

senior bond return (%)

π=0.05 π=0.10

Return distribution with different loan default correlations

bond default probability for $\beta=\sqrt{0.75}$

bond default probability for $\beta=\sqrt{0.25}$

senior bond return (%)

$\beta=\sqrt{0.25}$ $\beta=\sqrt{0.75}$

Both plots use baseline parameter $\beta = \sqrt{0.25}$.

Both plots use baseline parameter $\pi = 0.05$.

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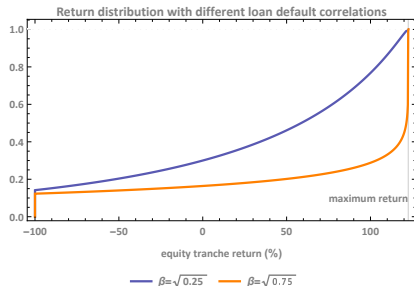
Risk analysis of equity tranche

- Increase in pool default rate decreases equity returns
    - Return distribution shifts to the *left*
- High default correlation increases equity returns
    - Return distribution shifts to the *right*
    - Equity has limited downside but unlimited upside
    - High correlation→high likelihood of very many and very few defaults
    - Former doesn't diminish expected return, since equity tranche value cannot go below zero, but latter adds to expected return
- Table displays *probabilities* of a loss on the equity tranche for different settings of pool loss distribution *parameters*

Equity tranche: probability of negative return			
	$\beta = \sqrt{0.05}$	$\beta = \sqrt{0.25}$	$\beta = \sqrt{0.50}$	$\beta = \sqrt{0.75}$
$\pi = 0.025$	0.0355	0.1241	0.1200	0.0899
$\pi = 0.05$	0.3458	0.3000	0.2328	0.1642
$\pi = 0.10$	0.8903	0.5801	0.4146	0.2884

Pool default behavior and equity tranche returns
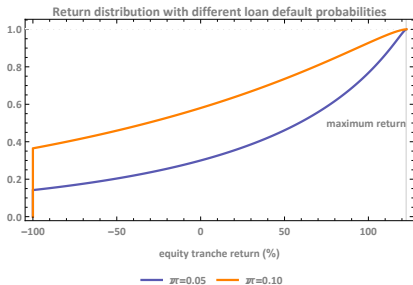
Return distribution with different loan default probabilities

equity tranche return (%)

—— π=0.05   —— π=0.10

maximum return

Return distribution with different loan default correlations

equity tranche return (%)

—— $\beta=\sqrt{0.25}$ —— $\beta=\sqrt{0.75}$

maximum return

Both plots use baseline parameter $\beta = \sqrt{0.25}$.

Both plots use baseline parameter $\pi = 0.05$.

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   └─ Securitization risk modeling
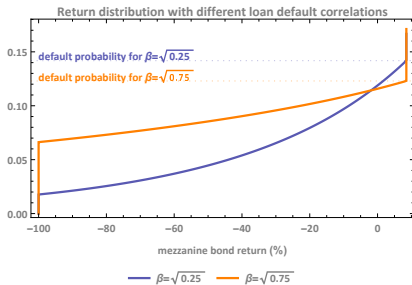
Risk analysis of mezzanine tranche

- Increase in pool default rate decreases return
- Impact of default correlation more ambiguous than for senior and equity
  - Will generally benefit less than equity and suffer less than senior from higher correlation
  - Depends heavily on attachment/detachment points
- Table displays default *probabilities* for different settings of pool loss distribution *parameters*

| Mezzanine bond default probability | | | |
$\beta = \sqrt{0.05}$	$\beta = \sqrt{0.25}$	$\beta = \sqrt{0.50}$	$\beta = \sqrt{0.75}$
$\pi = 0.025$ 0.0007	0.0443	0.0679	0.0638
$\pi = 0.05$ 0.0379	0.1418	0.1478	0.1230
$\pi = 0.10$ 0.4401	0.3648	0.2973	0.2295

Pool default behavior and mezzanine tranche returns
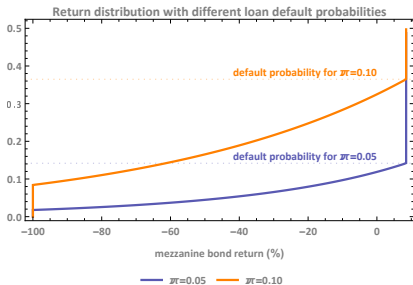
Return distribution with different loan default probabilities

default probability for $\overline{\pi}$=0.10

default probability for $\overline{\pi}$=0.05

mezzanine bond return (%)

—— $\overline{\pi}$=0.05   —— $\overline{\pi}$=0.10

Return distribution with different loan default correlations

default probability for $\beta=\sqrt{0.25}$

default probability for $\beta=\sqrt{0.75}$

mezzanine bond return (%)

—— $\beta=\sqrt{0.25}$ —— $\beta=\sqrt{0.75}$

Both plots use baseline parameter $\beta = \sqrt{0.25}$.

Both plots use baseline parameter $\pi = 0.05$.

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Risk modeling and structuring of securitizations

- Risk modeling used to structure a securitization
    - Attachment and detachment points, i.e. tranche sizes
    - Structure also affected by assessment of pool credit quality and cash flows
- **Example:** Suppose it is desired that senior bond have a default probability no greater than 1 percent
    - Find required attachment point a_s, given pool credit parameters
- Required attachment point satisfies

$$x^\circ = 1 - \frac{(1 - a_s)(1 + c_s)}{1 + r_l}$$
$$0.01 = 1 - \mathbf{P}\left[\tilde{x} \le x^\circ\right]$$

- Using baseline parameters, required attachment point is $a_s = 0.7552$
- Equity and/or mezzanine tranches will need to be somewhat wider

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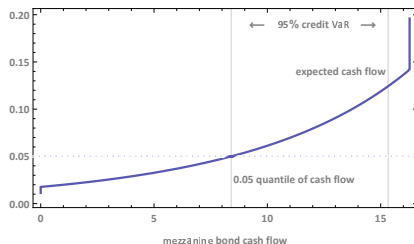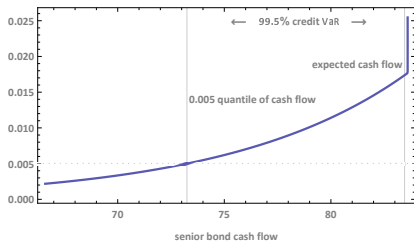   └─Securitization risk modeling

Ratings inflation

- **Ratings inflation:** assignment by rating agencies of unwarranted high ratings to bonds, particulalrly securitization tranches
- Can be achieved i.a. through
    - Setting attachment points so that senior tranches larger
    - Underestimate loan pool default probabilities, expected loss
- Motivation:
    - **"Issuer-pays":** rating agencies paid by issuers, who benefit from having lower-coupon senior bonds a larger share of liabilities
    - But investors also interested in higher ratings to satisfy regulatory constraints, increase available highly-rated issue volume

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Credit Value-at-Risk of a securitization

- Credit VaR can be computed using two components
    - Quantile of cash flow or return to any tranche using loss distribution
    - Expected loss (EL) as integral of product of loss density and cash flow or return
- Credit VaR equal to loss at a specified quantile minus EL

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Credit VaR of bond tranches

Vertical grid lines in each plot placed at cash flow with specified cumulative probability, and at expected value of cash flow. The distance between the two grid lines is the credit VaR for the tranche. Both plots use baseline parameters $\pi = 0.05$ and $\beta = \sqrt{0.25}$.

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Tail risk

- Low default probability but very high LGD
- Clusters of default: what if more likely?
- Much higher default probability than assumed
- Combination of ratings and capital standards

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 └─Securitization risk modeling

Correlation and convexity

- Market risk consequences of tail risk
- The role of default correlation
- Equity-AAA tradeoff
  - High correlation benefits equity, reduces value of AAA
  - Low correlation: high probability of steady trickle of defaults, unless default probability very low